Dual Role Platforms and Search Order Distortion *

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We investigate the impact of search order distortion, the act of steering consumers to search for a particular product first, by a vertically integrated platform that intermediates the products of a third-party seller as well as its own product. We show that the effects of search order distortion on prices and welfare depend on the search costs and commission rates. Even though a vertical separation policy could improve welfare through lower prices, a policy that only prohibits search order distortion could harm welfare and consumer surplus when commission rates are high. This result sheds lights on a potential risk of the policy that requires dominant platforms to be neutral while allowing them to sell on the marketplaces.

Keywords: consumer search, dual role platform, search order distortion, self-preferencing, structural separation

JEL classification: D11, D83, L13, L49

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1. Introduction

In recent decades, there has been a growing number of *dual role* platforms that not only provide a marketplace for third-party sellers but also act as a seller. For example, Amazon, Google, and JD provide e-commerce marketplaces to third-party sellers and charge commissions, while they also supply their own products on the same marketplace; Walmart and Target are currently rolling out their dual role platforms, both online and offline.

The expansion of such dual role platforms has given rise to various competition policy discussions, one of which relates to the conflict of interest and the *self-preferencing* behavior of platforms with a dual role. That is, they often favor their own products, not by competition, but by using the platform's advantageous position. For example, the EU Commission and the US House Judiciary Antitrust Subcommittee argued that Google directly manipulates search results and promotes its own content in search results, while it demotes competitors' content even if its own content is inferior.[1] Several studies, authorities, and journalists have suggested that Amazon uses its "Buy Box" function and search algorithms to steer consumers to its own products.[2]

As shown by these examples, dual role platforms often engage in this sort of self-preferencing behavior that distorts consumers' information search behavior. Specifically, they distort consumers' search order to promote a particular product, which is typically their own product.[3] Although there is a growing literature on dual role platform's self-preferencing behavior, no study incorporates the fact that self-preferencing behavior takes the form of search order distortion.[4] The purpose of this paper is to

[1] See http://ec.europa.eu/commission/presscorner/detail/en/IP_17_1784 (accessed Oct. 2020) and US House of Representatives Subcommittee on Antitrust (2020).

[2] See Angwin and Mattu (2016); Chen et al. (2016); Khan (2019) for examples of academic studies; see Hoppner and Westerhoff (2018) for the EU authorities' investigation, and a Wall Street Journal report available at http://www.wsj.com/articles/amazon-changed-search-algorithm-in-ways-that-boost-its-own-products-11568645345 (accessed Oct. 2020) on the use of search algorithms for steering consumers.

[3] Sometimes referred to as "own-content bias" (Wright, 2011; de Cornière and Taylor, 2019).

[4] For example, self-preferencing has been modeled as an increase in the probability that a consumer finds the platform's product when he/she collects information about products (Zennyo, 2021), when the platform only shows its own products (Hagiu et al., 2020), and under a biased purchase recommendation to uninformed consumers (de Cornière and Taylor, 2019).

explore the welfare properties of the dual role platform's search order distortions as one realistic representation of self-preferencing behavior and to examine the effectiveness of policies that restrict search order distortion by the dual role platform.

To this end, we develop a stylized model that captures both a dual role platform and search order distortion. To model the dual role platform, we consider a vertically integrated platform that obtains profits from its own products and commission revenues. To capture the search order distortion, we consider the model of sequential consumer search where consumers are directed to search for the dual role platform's product first. In other words, the platform's product is made prominent in the sense of Armstrong et al. (2009).

In our model, the monopolistic platform provides a marketplace. There are two sellers that sell products through the marketplace, and they must pay *ad valorem* commissions to the platform to complete the transaction in the marketplace. That is, sellers pay a portion of their revenue to the platform. The platform owns one of the sellers, from which it can earn more profit than other sellers. There is also a mass of consumers who are initially uninformed about the valuations of the products and the actual prices set by sellers. They must search for product information by visiting sellers on the platform and incur a search cost per firm. The platform distorts the consumer's search order by steering them to search for its own product first.

Using this framework, we characterize the equilibrium outcomes in which the platform steers consumers to search for its own product first. The prices of both the platform's product and that of the third-party seller depend on the commission rates and particularly increase with these rates. Furthermore, whether the platform sets a price lower than the third-party seller also depends on the commission rates. When the commission rate is sufficiently low, the profit from its own product is essential for the platform. Hence, the platform sets a lower price than another seller to attract consumers by taking advantage of a prominent position in the search order. However, when commission rates are not low, the profit from the other seller becomes equally important to the platform, and the platform has an incentive to set a higher price, making the platform's price higher than that of the third-party seller.

Based on the equilibrium characterization, we consider the policy implications of the search order distortion by comparing the baseline model with counterfactual scenarios where search order distortion is banned. Specifically, motivated by the policy proposals of scholars and politicians, we focus on policies regarding neutrality regulation and structural separation.[5]

We first compare the baseline model with the equilibrium in the case where search order is randomized, as a case where search order distortion is prohibited. We find that compared with this random search order scenario, search order distortion allows sellers to segment the market and weakens price competition when the commission rate is low. In such a case, search order distortion increases prices and reduces total surplus and consumer surplus, as observed by Armstrong et al. (2009). However, in the presence of commission revenue, another countervailing effect arises. With commission revenue, the platform internalizes its impact on the third-party seller, thereby increasing prices. Such a collusive effect is weaker under search order distortion than under the random search order scenario because the market is already segmented. Therefore, search order distortion tends to lower the prices and improve total surplus and consumer surplus when the commission rate is high. Furthermore, there is a case where search order distortion improves consumer surplus, and thus the platform has an incentive to introduce it. These results suggest that an unconditional prohibition of search order distortion may lower consumer surplus by increasing prices, and the competition authorities should be cautious about the competitive effects of banning search order distortion.

Next, we consider the case of structural separation where the platform and sellers are financially separated, and the platform is required to be neutral. Because the sellers have no incentive to set collusive prices under structural separation, the separation always lowers prices, and increases total surplus and consumer surplus. This implies that in terms of eliminating the anti-competitive effects of search order distortion, vertical separation would be a better alternative than banning only search order distortion.

The results of our study provide the following policy implication. Prohibiting search

[5] See US House of Representatives Subcommittee on Antitrust (2020) for the neutrality regulation and Elizabeth Warren's discussion of structural separation, https://medium.com/@teamwarren/heres-how-we-can-break-up-big-tech-9ad9e0da324c (accessed Oct. 2020).

order distortion may have an adverse effect even in the short-run price competition where the commission rates or investment levels of the platform are fixed. This sheds lights on a potential risk of the policy that requires dominant platforms to be neutral while allowing them to sell in the marketplaces, which contributes to the recent policy discussion on the regulation of dominant digital platforms.[6]

The novelty of our study lies in incorporating sequential consumer search in the analysis of the self-preferencing behavior of dual role platforms as a realistic representation of consumer behavior. By doing so, we show a novel interaction between the search order distortion and the revenue structure of the dual role platforms that is missed by many of the previous studies. In particular, most existing studies on dual role platforms conclude that regulating self-preferencing behavior improves consumer surplus, at least in the short-run, as long as it is effectively enforced, which is not the case in our framework for a wide range of environments.[7]

The remainder of the article proceeds as follows. In the remainder of this section, we describe related literature and our contributions to it. Section 2 describes the basic model. Section 3 characterizes the equilibrium outcomes when the platform steers consumers to search its own product first. Section 4 compares the outcomes of our baseline model to other models to obtain policy implications. We then conclude in Section 5. All proofs and calculations are in the appendix.

Related literature

This study is related to the literature on consumer search theory and studies on the dual role of platforms.

Our framework builds on the consumer search theory framework of ordered search. There is a large number of studies in which consumers search prices and valuations of goods in random order (Wolinsky, 1986; Anderson and Renault, 1999; Rhodes, 2014; Moraga-González et al., 2017), and a few studies have built a model in which

[6] See Khan (2019); Crémer et al. (2019); Alexiadis and de Streel (2020); Parker et al. (2020); Calvano and Polo (2021); Jullien and Sand-Zantman (2021) for examples of these policy discussions.

[7] See Hagiu et al. (2020); Zennyo (2021). One exception is de Cornière and Taylor (2019), where self-preferencing behavior can improve short-run consumer surplus when ad-financed sellers compete in qualities.

consumers search in a specific order, that is, an ordered search model (Armstrong et al., 2009; Zhou, 2011; Armstrong, 2017). Our study is closely related to the model of Armstrong et al. (2009) in which consumers search one "prominent" seller first. The search order distortion in our framework can be interpreted as the prominence of the platform-owned seller. Armstrong et al. (2009) show that sellers may prefer to be searched first and that making a firm prominent always reduces the welfare. The difference between their framework and ours is the presence of a monopolistic platform that serves a marketplace and also supplies a product by itself. Because of this difference, we show that making platform-owned seller prominent can improve the welfare and consumer surplus when the commission rate is high. Our contribution to the literature on consumer search is to show the novel interaction between the competitive effects of prominence and the revenue structure of the dual role platforms, a finding that can even overturn the results of previous studies.

Our study is also closely related to the growing literature on the dual role of platforms. This literature discusses topics such as a platform's decision to operate its first-party content (Hagiu and Spulber, 2013; Hagiu and Wright, 2015; Dryden et al., 2020; Padilla et al., 2020; Etro, 2021), and the welfare effects of self-preferencing behavior (de Cornière and Taylor, 2019; Hagiu et al., 2020; Zennyo, 2021).[8]

Our study belongs to this latter stream of the literature. de Cornière and Taylor (2019) explore the impact of the bias in the recommendation system on the competition between sellers and show that the bias may improve consumer surplus typically when sellers are ad-sponsored, whereas it reduces consumer surplus when sellers compete in prices. Hagiu et al. (2020) consider steering consumers to buy platform-owned seller's product as a form of self-preferencing and show that even though steering weakens competition, the platform's entry into the marketplace weakly improves consumer surplus and welfare. Zennyo (2021) considers a model in which consumers determine the number of products they sample, and the platform can place its own product in the search results. In his simultaneous search setting, even though self-preferencing lowers welfare for a given commission, it induces the platform to set lower commission

[8] See Gawer and Henderson (2007); Zhu and Liu (2018); Wen and Zhu (2019) for the empirical analysis of the effects and determinants of a platform's entry into a marketplace.

rates to attract consumer participation, making it possible to improve welfare. Our study differs from these studies in that we model self-preferencing as a favorable position in a sequential search environment and show that self-preferencing might be pro-competitive even in an environment where it would be anti-competitive when self-preferencing were modeled in other ways. Specifically, even though sellers compete in prices, and commission rates are exogenous, self-preferencing modeled as search order distortion can be pro-competitive in our setting.

2. The model

2.1. Setting

There is a marketplace intermediated by a monopolistic platform. In the marketplace, two sellers provide horizontally differentiated products. The platform owns one of the sellers, which is indexed by M. The other seller, which is indexed by N, is a third-party seller.

Consumers The consumer's search procedure is the same as that of Armstrong et al. (2009). There is a unit mass of consumers who wish to purchase one of the products in the marketplace. Consumers initially have imperfect information about the actual prices and the valuation of the products. They therefore have to gather the information sequentially. We assume that the first search is costless and the second search costs $s > 0$.[9] Following Perloff and Salop (1985) and the standard consumer search literature, we assume that each consumer has idiosyncratic tastes for each product. The valuation of the product from seller $i \in \{M, N\}$ is denoted by u_i, and we call the valuation the *match utility* of seller i. Specifically, the consumer's utility from purchasing product i after $l \in \{1, 2\}$ search is given by

$$U_{il} = u_i - p_i - (l - 1)s.$$

[9] The assumption that the first search is costless is a standard assumption in the literature and not essential for the results.

When each consumer visits a seller i, her match utility u_i for the product of the seller is independently and randomly drawn from a common cumulative distribution function $F(u)$, which is commonly known. For simplicity, we assume that u_i is uniformly distributed over $[0, 1]$, that is, $F(u) = u$ for all $u \in [0, 1]$. Each consumer's valuation is independent across consumers and sellers. We impose a free-recall assumption under which each consumer can freely go back to the sellers they have visited, which is generally imposed in the consumer search theory literature. Finally, we assume that there is no outside option, so the market is covered.[10]

In the baseline setting, we assume that consumers always search for seller M first. This is motivated by the possibility that the platform steers consumers to search for the platform-owned seller M first, and we call such a strategy a *search order distortion*. For example, a platform could adopt a strategy to show consumers its own content at the top of the search results.[11] In Section 4, we analyze another search order and compare the equilibrium results.

Sellers Sellers set prices and must pay the fraction $r \in (0, 1)$ of the revenue to the platform as ad valorem commissions. For ease of exposition, we assume that r is predetermined and exogenously given to the sellers. We also assume that the marginal costs of the platform and sellers are zero.

When seller i sells D_i units of products, it earns revenue $p_i D_i$. Then, seller N's profit is given by $\Pi_N = (1 - r)p_N D_N$, and the joint profit of seller M and the platform is given by $\Pi_M = p_M D_M + r p_N D_N$. Seller N sets p_N to maximize its own profit, and seller M sets p_M to maximize the joint profit.

Timing and equilibrium The timing of the game is as follows:

1. Given an exogenous commission rate r, sellers set prices (p_M, p_N) simultaneously and independently.

[10] Introducing an outside option does not change the qualitative results substantially. See the Online Appendix for the results when the market is not fully covered.

[11] As Armstrong et al. (2009) point out, we can also consider the case where the platform can steer a fraction of consumers α to search seller M first, while the remaining consumers search randomly as usual.

2. Without observing prices, consumers first search for seller M and observe p_M and u_M and choose whether to search for seller N by incurring search cost s.
    a) If a consumer searches seller N, she observes p_N and u_N and purchases a product from the seller i with higher $v_i = u_i - p_i$.
    b) If a consumer does not search seller N, she buys a product from seller M.

We adopt perfect Bayesian equilibrium as an equilibrium concept and focus on the equilibrium with passive beliefs, under which consumers do not alter their beliefs about prices after observing an out-of-equilibrium price.

Discussion on modeling assumption Before analyzing the model, we discuss several assumptions imposed in our framework. We assume that there is no outside option for consumers. This implies that consumers always join the platform and buy one of the products. We also assume that the commission rate r is exogenous. These assumptions are valid if we interpret the analysis as a short-run analysis where the platform has already set r and consumers have joined the platform. In the long run, consumer participation and the commission rate would be determined endogenously, which is abstracted away in our main analysis. In Section 4.3, we discuss the role of an outside option and how the long-run effects would differ from the short-run analysis.

We also assume that there is no marginal cost for sellers and platforms. If there is a marginal cost for sellers, an increase in the commission rate would increase the price level through an increase in the perceived marginal costs. However, in our setting, because an increase in marginal cost is directly reflected in the price, the presence of the marginal cost would not be essential in our analysis.

2.2. Optimal search rule

We first characterize an optimal search rule with the platform's search order distortion.[12] Specifically, after the initial search, a consumer compares the expected incremental benefit from the additional search with an additional search cost s.

[12] See Weitzman (1979) for a general treatment.

We first describe the optimal search rule in the simplest case where all sellers set the same prices, $p_M = p_N = p$. Define x to be the solution to the equation:

$$\int_x^{\infty} (u - x) dF(u) = \int_x^{\infty} [1 - F(u)]\, du = s.$$

Then, when $p_M = p_N = p$, each consumer's search rule is based on the constant reservation value x; she searches beyond seller M if $u_M < x$; otherwise, she stops searching and purchases from seller M immediately.

Now consider the optimal search with different prices. Let u_M be a consumer's match utility with seller M, and p_N^e be consumer's expectation of the price of seller N. We can show that each consumer will prefer to stop at seller M and purchase its product immediately if $u_M - p_M > x - p_N^e$. In our setting where u is uniformly distributed on the interval $[0, 1]$, we have

$$\int_x^1 (1 - u) du = s \Leftrightarrow x = 1 - \sqrt{2s}.$$

To guarantee that x is nonnegative, we assume that $s \leq 1/2$.

Based on this optimal search rule, we derive the demand for the sellers under search order distortion.

2.3. Demand with search order distortion

In this subsection, we describe the demand for each seller when the platform steers consumers to the specific seller.

Let 1 be the index of the seller that is searched first and 2 be that of the seller that is (potentially) searched second by consumers. Recall that, according to the optimal search rule, each consumer stops and purchases the product from seller 1 at price p_1 if $u_1 - p_1 \geq x - p_2^e$. Hence, each consumer immediately purchases from seller 1 with probability

$$1 - x - p_1 + p_2^e,$$

and hereafter we refer to this as *fresh demand* under search order distortion.

Each consumer may find it optimal to purchase the product from seller 1 even after searching for seller 2 when the utility from seller 2's product is small. We call such a demand *returning demand*, which is given by,

$$
\begin{aligned}
Pr(u_2 - p_2 < u_1 - p_1 < x - p_2^e) = &\int_0^{x-p_2^e+p_1} F(u_1 - p_1 + p_2)dF(u_1) \\
=&\frac{1}{2}(x + p_1 - p_2)^2 + (p_2 - p_1)(x + p_1 - p_2^e).
\end{aligned}
$$

Then, the demand for first-searched seller 1 is given by the sum of fresh demand and returning demand,

$$
d_1(p_1, p_2, p_2^e) \equiv 1 + \frac{(x - p_2^e + p_1)^2}{2} - (1 - p_2 + p_1)(x - p_2^e + p_1), \tag{1}
$$

and that for seller 2 is given by $d_2(p_1, p_2, p_2^e) \equiv 1 - d_1(p_1, p_2, p_2^e)$ because of the full coverage assumption.

To simplify the notation, we provide the following definitions and calculations. Let $\Delta_{21} \equiv p_2 - p_1$. In the equilibrium, $p_2^e = p_2$ holds by the consistency of the expectation. Then, the derivatives of the demand functions are

$$
\begin{aligned}
\frac{\partial d_1(p_1, p_2, p_2^e)}{\partial p_1} &= -(1 - \Delta_{21}) \equiv -\delta_1(\Delta_{21}), \\
\frac{\partial d_2(p_1, p_2, p_2^e)}{\partial p_2} &= -(x - \Delta_{21}) \equiv -\delta_2(\Delta_{21}),
\end{aligned} \tag{2}
$$

$\partial d_2/\partial p_1 = -\partial d_1/\partial p_1$, and $\partial d_1/\partial p_2 = -\partial d_2/\partial p_2$. Furthermore, in the equilibrium where $p_2 = p_2^e$, we have

$$
d_1(p_1, p_2, p_2) = 1 - \frac{(x - \Delta_{21})(2 - x - \Delta_{21})}{2} \equiv d(\Delta_{21}). \tag{3}
$$

In the following analysis, let $\Delta \equiv \Delta_{NM}$.

Note that from equation (2), $\delta_2(\Delta) > \delta_1(\Delta)$ for all Δ. This means that demand is less responsive to the changes in the price of seller 2, because that price is unobservable to consumers until they search. As we will see in the next section, this would induce seller 2 to set relatively higher prices unless the commission rate is sufficiently high.

3. Equilibrium under search order distortion

Now we characterize the equilibrium prices and profit when a platform steers consumers to its own product first.

3.1. Equilibrium prices

Here, we use the superscript "P" to represent the search order distortion in which the "platform"-owned seller's product is searched first. Then, using the expressions derived in the previous section, the platform's profit function can be written as $\Pi_M^P = p_M D_M^P + r p_N D_N^P$, where $D_M^P = d_1(p_M, p_N, p_N^e)$ and $D_N^P = 1 - D_M^P$. The profit of the third-party seller N is given by $\Pi_N^P = (1-r) p_N D_N^P$. Based on these profit functions, the equilibrium prices are given by the first-order conditions

$$\frac{\partial \Pi_M^P}{\partial p_M} = d(\Delta) - \delta_1(\Delta) p_M + r\delta_1(\Delta) p_N = 0, \tag{4}$$

$$\frac{\partial \Pi_N^P}{\partial p_N} = (1-r)\left[1 - d(\Delta) - \delta_2(\Delta) p_N\right] = 0. \tag{5}$$

Combining equations (4) and (5), we obtain the equilibrium condition for the price difference Δ. Let $\Delta^P(x,r)$ be the equilibrium price difference under search order distortion given x and r. Then, equation (5) can be used to compute the equilibrium prices p_N^P and $p_M^P = p_N^P - \Delta^P$.

This equilibrium characterization is valid when there are both types of consumers, namely those who search for seller M and those who do not, that is, $\Delta^P > x - 1$. The following lemma shows the explicit values of the equilibrium variables (Δ^P, p_M^P, p_N^P) and the condition under which these characterizations are valid.

Lemma 1. *For $r \leq \hat{r}^P(x) \equiv \frac{(11-4x)(1-x)}{(2-x)(3-2x)}$, the equilibrium price difference with search order distortion is given by*

$$\Delta^P(x,r) \equiv \frac{7 - 3r - x + xr - \sqrt{(7 - 3r - x + xr)^2 - 4(4-r)(x(1-x+r) - 2r)}}{2(4-r)}.$$

The equilibrium prices with search order distortion are given by

$$p_N^P(x,r) = \frac{2 - x - \Delta^P}{2},$$
$$p_M^P(x,r) = \frac{2 - x - 3\Delta^P}{2}.$$

Based on this equilibrium characterization, we analyze the behavior of equilibrium prices. In particular, we consider the effect of commission rate r on the equilibrium prices.

There are two key mechanisms that determine the prices set by sellers in the presence of the dual role platform and search order distortion. First, a dual role platform tends to set a higher price because the platform's profit depends not only on the revenue from its product but also on that of the other seller. We call this the *dual role* effect, which makes the platform-owned seller's price relatively high. By contrast, the presence of search order distortion induces the platform to set a relatively lower price to attract more "fresh" demand, whereas it also induces the third-party seller that has no fresh demand to set a higher price. We call this the *search order* effect, which makes platform-owned seller's price relatively low. Note that the search order effect is always dominant when $r = 0$, which is shown by Armstrong et al. (2009).

Which of these effects will be dominant depends on r. The size of the dual role effect increases in r, while the search order effect does not depend on r. Therefore, the platform-owned seller sets the price higher than that of a third-party seller if and only if r is above a certain critical value.

Finally, the dual role effect affects not only price ranking but also price levels. Because the platform-owned seller sets a higher price, the third-party seller also sets a higher price through strategic complementarity, although it increases at a lower rate than the price of the platform-owned seller. Thus, both prices increase in r

The following proposition summarizes the above discussion.

Proposition 1. *In the equilibrium with search order distortion, the following statements hold true.*

1. Δ^P *decreases in* r.

2. *Seller M sets a higher (resp. lower) price than seller N for $r > \bar{r}$ (resp. $r < \bar{r}$), where $\bar{r} = \frac{(1-x)x}{(2-x)}$.*

3. *Both p_M^P and p_N^P increase in r.*

Table 1 shows a numerical example of $\bar{r}$ such that $p_M = p_N$ in Proposition 1, and Figure 1 illustrates the behavior of equilibrium prices.

Table 1: Numerical example of $\bar{r}$.

	$x = 0.6$ ($s = 0.08$)	$x = 0.9$ ($s = 0.005$)	Max ($s = 0.086$)
$\bar{r}$	0.171	0.082	0.172

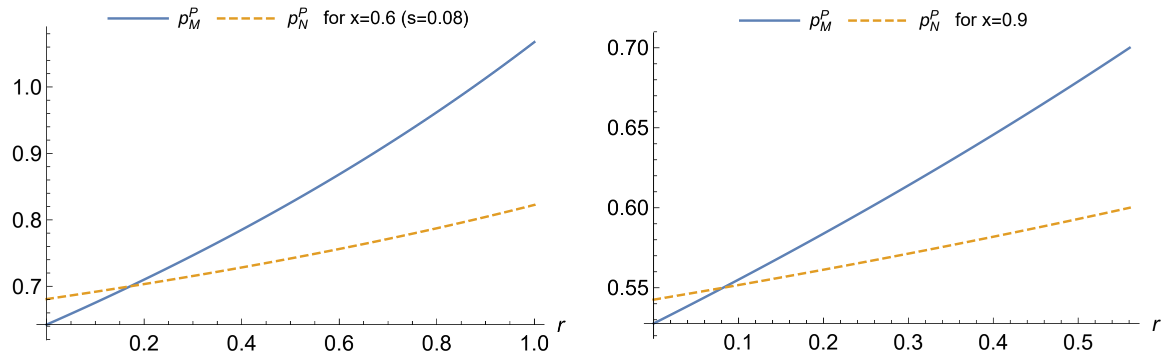

Figure 1: Prices for $x = 0.6$ ($s = 0.08$) and for $x = 0.9$ ($s = 0.005$).

These results contrast with that of Armstrong et al. (2009), who show that p_M is always lower than p_N. As in the numerical example in Table 1, $\bar{r}$ is not very large, especially for small s. For example, Amazon sets commission rates of about 8%–15% for most of the categories of items, which suggests that $r > \bar{r}$ may hold in practice, depending on the actual search costs.[13] Hence, $p_M > p_N$ may hold under plausible conditions. In Armstrong et al. (2009), a prominent seller that is searched first (seller M in our model) always has an incentive to set the lowest price to attract more fresh demand. In our setting, the presence of commission revenue makes the platform-owned seller set a higher price than the other sellers.

[13] See http://sell.amazon.com/pricing.html (accessed Oct. 2020).

4. Impact of search order distortion

So far, we have characterized the outcomes when the platform distorts consumers' search order. Such a search order distortion is often considered to be an unfair platform design. As many studies and committees have noted (Khan, 2019; Parker et al., 2020; Alexiadis and de Streel, 2020; Cabral et al., 2021), a platform's “fairness" is the key element in considering competition policy. As a result, several regulatory instruments such as neutrality regulation and vertical separation are proposed as remedies to unfair practices.

In this section, we compare the outcomes of the baseline setting with other market scenarios that are achievable using such policy instruments and examine the effectiveness of these policies. Specifically, we consider two scenarios: a random search order scenario with a vertically integrated seller and a random search scenario with vertically separated sellers. The former would be achieved by a neutrality regulation that prohibits search order distortion, and the latter would be achieved by vertical separation. Here, we consider the random search order as a “neutral" scenario because the random search order treats similar sellers similarly in terms of search priority.

4.1. Prohibition of search order distortion

Suppose the platform owns a seller but cannot distort the search order according to “non-discriminatory" treatment, and therefore the consumers' search order is randomized. Hereafter we use superscript "R" to represent the random order search scenario.

Equilibrium analysis Under this scenario, each seller i is searched first with probability $1/2$. Thus, using the notation of equation (1), the demand for each seller i is given by

$$D_i^R = \frac{1}{2}\left[d_1(p_i, p_j, p_j^e) + 1 - d_1(p_j, p_i, p_i^e)\right].$$

The platform's objective function is $\Pi_M^R = p_M D_M^R + r p_N D_N^R$ and seller N's objective function is $\Pi_N^R = (1-r)p_N D_N^R$. The first-order conditions are given by,

$$\frac{\partial \Pi_M^R}{\partial p_M} = \frac{d(\Delta) + 1 - d(-\Delta)}{2} - \frac{\delta_1(\Delta) + \delta_2(-\Delta)}{2} p_M + r \frac{\delta_1(\Delta) + \delta_2(-\Delta)}{2} p_N = 0, \tag{6}$$

$$\frac{\partial \Pi_N^R}{\partial p_N} = (1-r) \left[\frac{d(-\Delta) + 1 - d(\Delta)}{2} - \frac{\delta_1(-\Delta) + \delta_2(\Delta)}{2} \right] = 0. \tag{7}$$

Combining these equations, we obtain the equilibrium condition for the price difference Δ^R. Again, the first-order conditions are valid only when there are both types of consumers, those who search for the second product and those who do not, which turns out to be the case when $r < \hat{r}^R(x) = (5+x)(1-x)/[2(2-x)]$. Then, the following lemma characterizes the equilibrium price difference and the price levels.

Lemma 2. *For $r < \hat{r}^R(x)$, the equilibrium price difference with random consumer search is given by*

$$\Delta^R \equiv -\frac{r}{5 - 2r + x} \leq 0.$$

The equilibrium prices are given by

$$p_N^R(x,r) = \frac{1}{1+x} \frac{5+x}{5+x-2r},$$
$$p_M^R(x,r) = p_N^R - \Delta^R.$$

Furthermore, (i) both p_M^R and p_N^R increase in r; (ii) $p_M^R > p_N^R$ for any $r > 0$; and (iii) the price difference Δ^R decreases in r and increases in x.

There is one noticeable feature of the equilibrium prices under random search order. In contrast to the case of search order distortion, the price of the platform-owned seller's product is always higher than that of the third-party seller, which is observed from $\Delta^R \leq 0$. Because the only difference between the two sellers is the commission revenue of the platform-owned seller, this seller sets a higher price than the third-party seller so as to obtain revenue from the other seller. Figure 2 shows an example of equilibrium prices under a random search scenario.

Based on this equilibrium characterization, we compare the outcomes of the random order scenario with our baseline model of search order distortion.

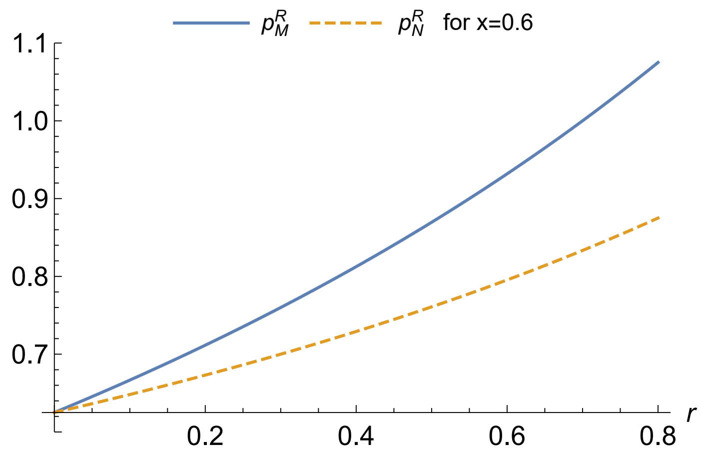

Figure 2: Prices in the random order search scenario ($s = 0.005$).

Effects on prices We first examine the impact of the prohibition of search order distortion on prices.

There are two countervailing effects of search order distortion on the equilibrium prices. First, under search order distortion, consumers who have an incentive to search the third-party seller should have a relatively low evaluation of the platform's product. In such a case, the impact of platform-owned seller's price on the demand for the third-party seller is relatively small. That is, the market is segmented. Therefore, the competition between the platform and the third-party seller weakens under search order distortion. Without commission, this effect is always dominant, and search order distortion raises prices.[14]

However, in the presence of commission revenue, there is an additional collusive effect in which the platform-owned seller cares about the revenue collected from the third-party seller. As we discussed above, under search order distortion, the consumers are segmented, and the externality in pricing decisions is smaller because of the weak substitution between products. Then, the collusive effect of an increase in r is weaker under search order distortion than under random search. Therefore, the platform may set a lower price under search order distortion than under random search when the commission rate is high. Formally, the best-response function of the platform-owned seller under random search shifts upward under r more than it does under search order distortion, which is shown in Appendix B.3.

[14] Rhodes and Zhou (2019) find a similar effect in the analysis of retail market structure.

In sum, search order distortion itself has anti-competitive effects of increasing the prices of both sellers, but once it is combined with the presence of commission revenue, the search order distortion has some pro-competitive effects because it mitigates the collusive effects of commission revenues. The following proposition summarizes this argument.

Proposition 2. *The following statements hold true:*

1. *At $r = 0$, search order distortion raises the prices of both products. Specifically, $p_N^P \geq p_M^P > p_N^R = p_M^R$ holds for all $x < 1$.*
2. *When $x > \bar{x} \equiv (7 - \sqrt{33})/2 \approx 0.628$, there exists $r^P(x) < \bar{r}$ such that $p_M^R > p_M^P$ holds for all $r \in [r^P(x), \bar{r}]$.*

Figure 3 shows the behavior of the equilibrium prices under the two scenarios. The prices under search order distortion are relatively lower when r is large. Furthermore, this figure indicates that for a sufficiently high commission rate, even the equilibrium price of the third-party seller under search order distortion may be lower than that under random search.

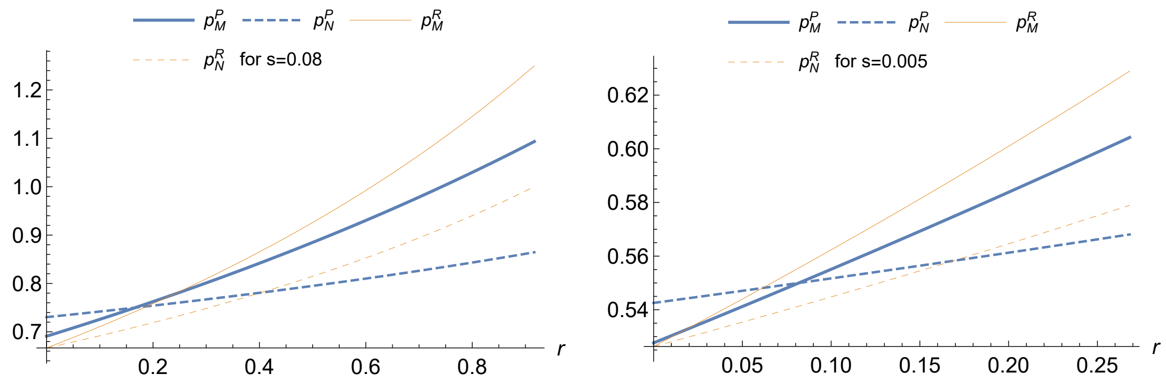

Figure 3: Prices for $x = 0.6$ ($s = 0.08$) and $x = 0.9$ ($s = 0.005$) (r is truncated at $\min\{\hat{r}^P, \hat{r}^R\}$).

The comparison of the pricing behaviors under the two scenarios indicates that prohibiting search order distortion may have the adverse effect of increasing the prices. In the following sections, we examine how total surplus and consumer surplus change when search order distortion is prohibited.

Effects on total surplus Here, we consider the impact of search order distortion on total surplus. In the setting with a covered market, total surplus is affected by whether the purchase of two products is made efficiently, which is determined solely by the price difference Δ. For any given price difference Δ, let $TS^P(\Delta, x)$ and $TS^R(\Delta, x)$ be the equilibrium total surplus. In Appendix B.1, we show that total surplus, as functions of (Δ, x), are given by

$$TS^P(\Delta, x) = \frac{1}{2} + \frac{x^2(3-2x)}{6} - \frac{\Delta^2(1+2x)}{6} + \frac{\Delta(\Delta^2 - x + x^2)}{3},$$
$$TS^R(\Delta, x) = \frac{1}{2} + \frac{x^2(3-2x)}{6} - \frac{\Delta^2(1+2x)}{6}.$$

The direct comparison shows that $TS^P(0, x) = TS^R(0, x)$, $\partial TS^P(\Delta, x)\partial\Delta < 0$ for all $\Delta \in (0, x)$, and $\partial TS^R(\Delta, x)\partial\Delta > 0$ for all $\Delta < 0$.

As we have seen in Proposition 1 and Lemma 2, search order distortion gives rise to a significant price difference $\Delta^P > 0$ when r is sufficiently small, while such a price difference is absent under the case of random search order. This implies that when r is small, the search order distortion reduces total surplus. However, as r grows, Δ^P approaches 0 as long as $r < \bar{r}$, whereas $|\Delta^R|$ continues to increase. At the point around $r = \bar{r}$, Δ^P becomes sufficiently small, whereas $|\Delta^R|$ is bounded away from 0. This implies that when r is close to $\bar{r}$, search order distortion improves total surplus. These observations suggest that in the range between $r = 0$ and $r = \bar{r}$, there exists a critical value r^{TS} such that search order distortion improves the total surplus if and only if $r > r^{TS}$, which is formally shown in the following proposition.

Proposition 3. *For any $x \in (0, 1)$, there exists a critical value of the commission rate $r^{TS} \in (0, \bar{r})$ such that $TS^P(\Delta^P, x) < TS^R(\Delta^R, x)$ for all $r \in [0, r^{TS})$, and $TS^P(\Delta^P, x) > TS^R(\Delta^R, x)$ for all $r \in (r^{TS}, \bar{r}]$.*

In sum, search order distortion can improve total surplus when the commission rate is high. Figure 4 illustrates the condition under which search order distortion improves total surplus. Note that the welfare result for $r = 0$ is obtained by Armstrong et al. (2009) in a more general setting with more than one third-party seller and outside options. The present paper is the first to show that the welfare ranking is reversed when r takes high values.

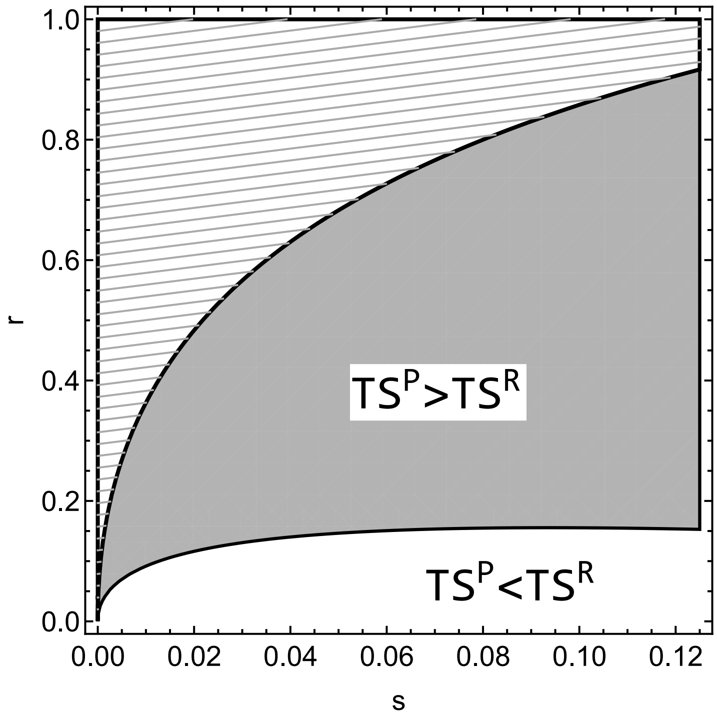

Figure 4: The gray area indicates that the search order distortion improves total surplus. The shaded area does not satisfy the conditions for $\hat{r}^P$ or $\hat{r}^R$.

Effects on consumer surplus Even though search order distortion can improve total surplus through the changes in allocative efficiency, what matters for many competition authorities is consumer surplus, which depends not only on the price differences but also on the price levels. Therefore, we next examine the impact of search order distortion on consumer surplus. In Appendix B.2, we show that consumer surplus under search order distortion CS^P and that under random search order CS^R are given by

$$CS^P(\Delta, p_M, x) = \frac{1}{2} - \frac{(1-x)^3}{6} - \frac{(1-x)^2 x}{2} + \frac{(1-x)^2 \Delta}{2} + \frac{(1-\Delta)^3}{6} - p_M,$$

and

$$CS^R(\Delta, p_M, x) = \frac{1}{2} - \frac{(1-x)^3}{6} - \frac{(1-x)^2 x}{2} + \frac{(1-\Delta)^3}{12} + \frac{(1+\Delta)^3}{12} - p_M - \frac{\Delta}{2},$$

respectively. We can see that both consumer surpluses depend not only on the price difference Δ but also on the price level p_M.

We first consider the case where r is sufficiently small. As we observed in Proposition 2, search order distortion raises the prices of both products when r is small. This suggests that when r is sufficiently small, search order distortion reduces consumer surplus. The following proposition confirms this observation.

Proposition 4. *When r is sufficiently small, search order distortion lowers consumer surplus, that is, $CS^P(x,0) < CS^R(x,0)$ for all $x \in (0,1)$.*

Proposition 2 also suggests that with large r, search order distortion may improve the consumer surplus because the prices under search order distortion may be lower than those under random search. Although deriving the condition under which search order distortion improves consumer surplus is complicated, we can derive such a condition in a limit case where the search cost is sufficiently small.

To analyze the case where search costs are small, suppose that $x = 1 - \epsilon$ and $r = \theta\epsilon$ for sufficiently small $\epsilon > 0$ and $\theta \in (0,3]$.[15] Let $\widetilde{CS}^P(\epsilon,\theta)$, $\widetilde{CS}^R(\epsilon,\theta)$ be the equilibrium consumer surplus under search order distortion and random search, respectively. Then, we have $\widetilde{CS}^P(0,\theta) = \widetilde{CS}^R(0,\theta)$ for any θ.

Based on this notation, consider the effect of search order distortion on consumer surplus. Note that when $\epsilon = 0$, both sellers produce $1/2$. Furthermore, the Roy's identity implies that

$$\widetilde{CS}^k(\epsilon,\theta) = \widetilde{CS}^k(0,\theta) - \sum_{i=M,N} D_i^k \left(-\frac{\partial p_i^k}{\partial x} + \frac{\partial p_i^k}{\partial r} \right) + o(\epsilon)$$

for $k \in \{P, R\}$. Evaluating these expressions for small ϵ for two search order scenarios and comparing them yield the following result on the impact of search order distortion on consumer surplus.

Proposition 5. *Suppose that $x = 1 - \epsilon$ and $r = \theta\epsilon$. Then if $\theta > 1$ (resp. $\theta < 1$), there exists $\underline{\epsilon} > 0$ such that compared with the random order search scenario, the search order distortion improves (resp. reduces) consumer surplus for all $\epsilon < \underline{\epsilon}$.*

[15] $\theta \leq 3$ is imposed because we have $\min\{\hat{r}^P(x), \hat{r}^R(x)\} = 3\epsilon + o(\epsilon)$ for sufficiently small ϵ.

As we have observed in Proposition 2, the equilibrium price of the platform-owned seller under search order distortion is lower than that under random search when search cost is small and the commission rate is high. In the limit case where search cost is close to zero, we obtain the analytical result that the price of the platform-owned seller under search order distortion is low enough to benefit consumers.

However, when s is not small, we cannot obtain a clear-cut analytical result. Instead, we present a numerical analysis to show the case where search order distortion improves consumer surplus. Figure 5 shows the conditions under which search order distortion hurts/improves consumer surplus.

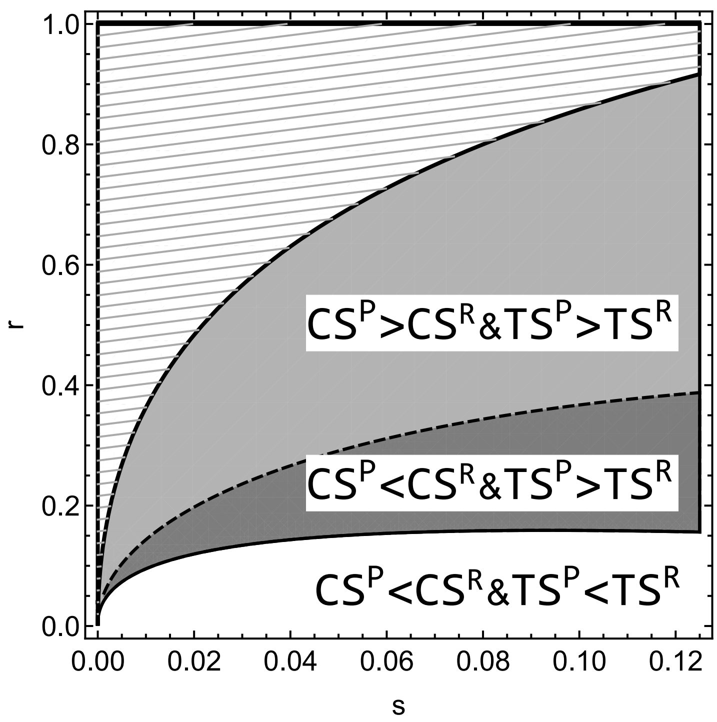

Figure 5: The light gray area indicates that search order distortion improves consumer surplus, while the dark gray and white areas indicate the opposite.

The platform's incentive to introduce search order distortion Finally, we briefly examine the platform's incentive to introduce search order distortion. Whether the prohibition of search order distortion has a relevant adverse effect on consumers depends on whether (i) the platform has an incentive to introduce search order distortion, and (ii) consumer surplus is greater under search order distortion than under random search. The following proposition shows that when search cost is small, the platform always has the incentive to introduce search order distortion.

Proposition 6. *Suppose that* $x = 1 - \epsilon$ *and* $r = \theta\epsilon$ *for* $\theta \in [0, 3)$. *Then, for sufficiently small* $\epsilon > 0$, *the platform's equilibrium profit under search order distortion is always greater than that when search order distortion is prohibited.*

This result, in conjunction with Proposition 5, provides the following policy implication. When the search cost is small and the commission rate is high, the platform has an incentive to introduce search order distortion, which is welfare superior to random search. When search order distortion is prohibited in such a case, consumer surplus decreases.

Although Proposition 6 is limited to the case with small search costs, we can numerically confirm that the platform has an incentive to introduce search order distortion, which benefits consumers even with larger search costs. Figure 6 shows the condition under which search order distortion increases the platform's equilibrium profit. We can observe that there is a region where search order distortion improves both the platform's equilibrium profit and consumer surplus. Thus, prohibiting search order distortion would reduce consumer surplus in such cases.

Summary Overall, compared with the random search order scenario, search order distortion increases prices and reduces total surplus and consumer surplus in the absence of commission revenues. However, when the commission rate is high, search order distortion tends to lower prices and improve total surplus and consumer surplus. Furthermore, there is a case where search order distortion improves consumer surplus, and the platform has an incentive to introduce it. Therefore, an unconditional prohibition of search order distortion may lower consumer surplus by increasing prices, and therefore the competition authorities should be cautious about the competitive effects of banning search order distortion.

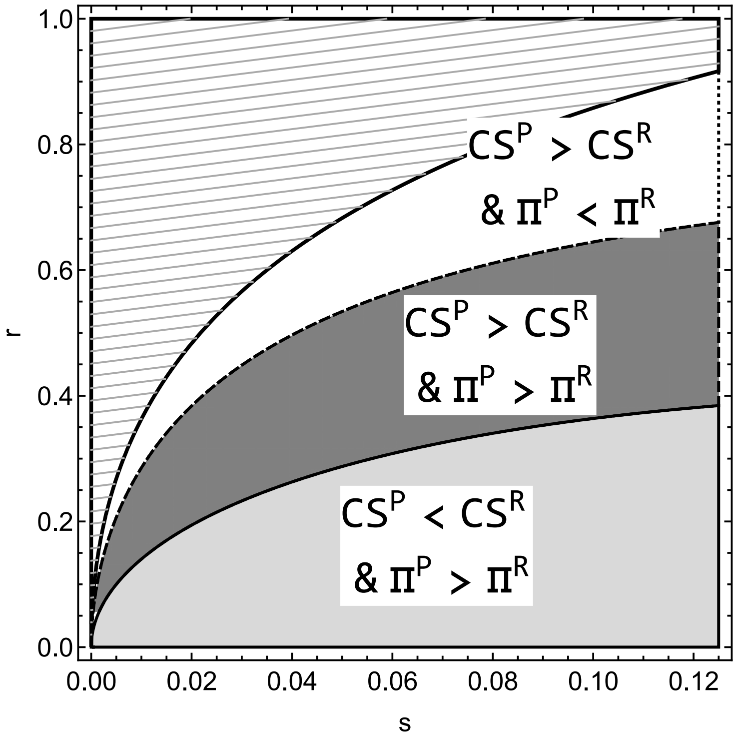

Figure 6: The shaded area does not satisfy the conditions for $\hat{r}^P$ or $\hat{r}^R$.

4.2. Vertical separation

We now consider vertical separation as another policy alternative. Vertical separation requires that the platform and sellers are financially separated. Specifically, suppose that there are one platform and two symmetric sellers, and the platform's profit is independent of any seller except for their commission revenues. We also assume that a uniform revenue-sharing rule r is exogenously given, and the platform randomizes the search order. Because all sellers are symmetric, and the revenue-sharing rule r is common and given to all sellers before the pricing decision, the platform's presence does not affect seller pricing. We focus on the equilibrium that all sellers set the same price. We suppose that each consumer has a passive belief that all sellers set the same prices. Note that this is a special case of the model in Section 4.1 where $r = 0$ for the random search order scenario.

In this case, there exists an equilibrium such that each seller sets the same price p_0, which is given by

$$p_0 = p^R(x, 0) = \frac{1}{1 + x}. \tag{8}$$

Let CS_0 and TS_0 be consumer and total surplus under vertical separation, respectively. Using Propositions 2, 3, and 4, we have the following result on the price and welfare effects of vertical separation.

Corollary 1. *Vertical separation lowers prices, strictly improves consumer surplus, and strictly improves total surplus for almost every r. Specifically,*

1. $p_M^P > p_0$ *and* $p_N^P > p_0$ *hold for any* $r > 0$,
2. $CS_0 > CS^P$ *for all* r, *and*
3. $TS_0 \geq TS^P$ *for all* r, *and* $TS_0 > TS^P$ *for any* $r \neq \bar{r}$.

Corollary 1 shows that the vertical separation policy achieves lower prices, greater consumer surplus, and greater total surplus. The logic behind Corollary 1-1 is similar to Proposition 2 for the case of low commission rates. When $r = 0$, the equilibrium prices under random search order are lower than those under search order distortion. Furthermore, because the equilibrium prices under search order distortion increase in r, they are always higher than those under vertical separation. The intuitions of Corollary 1-2 and 1-3 similarly follow those of Propositions 3 and 4. Vertical separation eliminates the price differences and lowers price levels, which are the source of allocative inefficiency and the reduction in consumer surplus.

4.3. Discussion

Finally, we discuss the potential elements that might affect our results but are not included in our framework.

We assumed that there is neither an outside option nor a consumer participation decision and that the commission rate r is exogenously given. If there is an outside option, each seller faces competition with not only the other seller but also with the outside option. This lowers the relative importance of the collusive effect of the commission rate, which makes search order distortion welfare superior. Thus, depending on the value of an outside option relative to that of the products of sellers, search order distortion may be less likely to improve welfare. In the Online Appendix, we show that although the presence of an outside option makes search order distortion less likely to improve welfare, the qualitative welfare properties of search order distortion remain valid.

When there is elastic consumer participation, the platform will choose r to balance consumer participation and per-consumer profit. Then, given that search order distortion is likely to achieve higher per-consumer profit, the platform will set a lower commission rate r to attract more consumers, as discussed by Zennyo (2021). If this is the case, incorporating consumer participation and an endogenous commission rate would favor search order distortion and make vertical separation less attractive in terms of welfare.

We also assumed that there is no value creation by the platform that may positively affect consumers and sellers.[16] The incentive for such value creation might be greater under vertical integration than under vertical separation. Again, this type of element would favor vertical integration over vertical separation.

Inclusion of the elements that have been omitted in our framework would not alter the main conclusion that prohibiting search order distortion might have an adverse effect, but may alter the result that vertical separation is the best policy alternative to deal with dual role platforms. Therefore, we need to be cautious about the long-run welfare properties of vertical separation.

5. Concluding remarks

In this study, we investigated the impact of search order distortion by a dual role platform using the framework of sequential consumer search theory. Using this framework, we compared the equilibrium under search order distortion with the two other scenarios: a scenario where only search order distortion is prohibited, and a structural separation scenario where all sellers are independent of the platform. We showed that compared with the case where only search order distortion is prohibited, search order distortion may lower the prices and improve total surplus and consumer surplus when the commission rate is high. By contrast, vertical separation always lowers prices and improves welfare. These results suggest that prohibiting only search order distortion

[16] See Kraemer and Zierke (2020); Gilbert (2021) for examples.

in the presence of a vertically integrated platform may have an adverse effect, whereas vertical separation can eliminate the anti-competitive effects of search order distortion.

There are some caveats to the interpretation of our results. First, we fixed several important strategic variables such as commission rates, investments, and marketing efforts. Incorporating these aspects may make vertical separation welfare inferior. In this regard, our results should be viewed as an assessment of the short-run effects of search order distortion.

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A. Proofs

A.1. Proof of Lemma 1

Combining equations (4) and (5), we obtain the equilibrium condition for the price difference Δ, which is given by

$$\begin{aligned} f(\Delta, x, r) &= 1 - \frac{(x-\Delta)(2-x-\Delta)}{2} + (1-\Delta)\Delta - (1-\Delta)(1-r)\frac{2-x-\Delta}{2} \\ &= 0. \end{aligned} \tag{A.1}$$

Noting that

$$\frac{\partial^2 f}{\partial \Delta^2} = r - 4 < 0,$$

we have the unique solution for $\Delta \in [x-1, x)$ that is consistent with first-order conditions if

$$f(x-1, x, r) = \frac{1}{2}\left[2x - 1 - 2(2-x)(1-x) - (2-x)(1-r)(3-2x)\right] \leq 0,$$

because $f(x,x,r) = 1 + (1-x)x - (1-x)^2(1-r) > 0$. Furthermore, $f(x-1,x,r) < 0$ holds if and only if

$$(1-r) \geq \frac{2x - 1 - 2(2-x)(1-x)}{(2-x)(3-2x)} = \frac{8x - 5 - 2x^2}{(2-x)(3-2x)},$$

which can be rewritten as

$$r \leq 1 - \frac{8x - 5 - 2x^2}{(2-x)(3-2x)} = \frac{(11-4x)(1-x)}{(2-x)(3-2x)} \equiv \hat{r}^P(x).$$

Then, $f(\Delta, x, r)$ can be rewritten as $f(\Delta, x, r) = g(\Delta, x, r)/2$, where

$$\begin{aligned} g(\Delta, x, r) &= 2 - (x - \Delta)(2 - x - \Delta) + 2(1-\Delta)\Delta - (1-\Delta)(1-r)(2 - x - \Delta) \\ &= [-4 + r]\Delta^2 + (7 - 3r - x + rx)\Delta + [-x(1 - x + r) + 2r]. \end{aligned}$$

Thus, Δ^P has the closed-form solution that is given as the solution to the quadratic equation $g(\Delta^P, x, r) = 0$:

$$\Delta^P(x,r) = \frac{7 - 3r - x + xr - \sqrt{(7 - 3r - x + xr)^2 - 4(4-r)[x(1-x+r) - 2r]}}{2(4-r)}.$$

□

A.2. Proof of Proposition 1

We first consider the effect of a change in r on the price difference Δ^P. Let

$$\begin{aligned} A(r) &= 4 - r, \\ B(r) &= 7 - 3r - x + xr, \\ C(r) &= x(1 - x + r) - 2r. \end{aligned}$$

Then, we have

$$\Delta^P(x,r) = \frac{B - \sqrt{B^2 - 4AC}}{2A},$$

$$\begin{aligned} A'(r) &= -1, \\ B'(r) &= -3 + x, \\ C'(r) &= -2 + x. \end{aligned}$$

$\Delta^P(x,r)$ has the derivative

$$\begin{aligned}
\frac{\partial \Delta^P(x,r)}{\partial r} &= -\frac{A'(r)}{A(r)}\Delta^P(x,r) + \frac{B'(r)}{2A} - \frac{1}{2A}\frac{2B'(r)B(r) - 4[A'(r)C(r) + A(r)C'(r)]}{2\sqrt{B^2 - 4AC}} \\
&= \frac{B(r) - \sqrt{B^2 - 4AC}}{2(4-r)^2} - \frac{3-x}{2(4-r)} + \frac{(3-x)B(r) - 2[C(r) + (2-x)A(r)]}{2(4-r)\sqrt{B^2 - 4AC}} \\
&= \frac{\Delta^P(x,r)}{A(r)} + \frac{(3-x)\Delta^P(x,r)}{\sqrt{B^2-4AC}} - \frac{C(r) + (2-x)A(r)}{(4-r)\sqrt{B^2-4AC}} \\
&= \frac{\left[B(r)\sqrt{X} - X + A(r)(3-x)(B(r) - \sqrt{X}) - 2A(r)[C(r) + (2-x)A(r)]\right]}{2(4-r)^2\sqrt{X}},
\end{aligned}$$

where $X = B^2 - 4AC$. The above expression is negative because the numerator can be written as

$$\begin{aligned}
&(7 - 3r - x + xr - (4-r)(3-x))\sqrt{X} - X + (4-r)(3-x)(7 - 3r - x + xr) \\
&- 2(4-r)[x(1-x+r) - 2r + (2-x)(4-r)] \\
=& -(5-3x)\sqrt{X} + (5-3x)(7 - 3r - x + xr) - 2(4-r)[8 - 5x + x^2] \\
=& -(5-3x)\sqrt{X} - (29 - 14x + 13x^2 - r - 4xr + x^2 r) < 0.
\end{aligned}$$

Now consider the effect of a change in r on each price. Recalling that,

$$p_N^P = \frac{2 - x - \Delta^P}{2}, \text{ and } p_M^P = \frac{2 - x - 3\Delta^P}{2}. \tag{A.2}$$

Since Δ^P is decreasing in r, both p_N^P and p_M^P are increasing in r. Furthermore, (A.2) means that p_M^P increases in r more than p_N^P.

Finally, we can directly verify that when $r = \bar{r} \equiv x(1-x)/(2-x)$, $\Delta^P(x,\bar{r}) = 0$ holds. □

A.3. Proof of Lemma 2

Equations (6) and (7) can be rewritten as

$$0 = \frac{1}{2}\left[1 - \frac{(x-\Delta)(2-x-\Delta)}{2} + \frac{(x+\Delta)(2-x+\Delta)}{2}\right] - \frac{1+x}{2}(p_M - rp_N), \tag{A.3}$$

$$0 = \frac{1}{2}\left[1 + \frac{(x-\Delta)(2-x-\Delta)}{2} - \frac{(x+\Delta)(2-x+\Delta)}{2}\right] - \frac{1+x}{2}p_N. \tag{A.4}$$

The first FOC subtracted by the second FOC is given by

$$\begin{aligned}
&-\frac{(x-\Delta)(2-x-\Delta)}{2}+\frac{(x+\Delta)(2-x+\Delta)}{2}+\frac{1+x}{2}\Delta+\frac{1+x}{2}rp_N \\
=&\frac{1}{2}\left[(2-x)(x+\Delta-x+\Delta)+\Delta(x+\Delta+x-\Delta)\right]+\frac{1+x}{2}\Delta+\frac{1+x}{2}rp_N \\
=&2\Delta+\frac{1+x}{2}\Delta+\frac{1+x}{2}rp_N=0.
\end{aligned}$$

Thus, when $r>0$, we have an expression for equilibrium p_N by

$$p_N=\frac{-\Delta}{r}\left(\frac{5+x}{1+x}\right). \tag{A.5}$$

Plugging the above expression into the second FOC (A.4), we obtain

$$\frac{1}{2}-\Delta+\frac{5+x}{2}\frac{\Delta}{r}=0.$$

Putting these together, we obtain the equilibrium condition for Δ:

$$\Delta=\Delta^R(x,r)\equiv-\frac{r}{5+x-2r}.$$

Hence, as long as $x-\Delta^R\leq 1$, that is

$$1+\Delta^R-x=\frac{(5+x)(1-x)-2(1-x)r-2r}{5+x-2r}\geq 0,$$

which is equivalent to

$$r\leq\hat{r}^R(x)\equiv\frac{(5+x)(1-x)}{2(2-x)}.$$

Δ^R is the equilibrium price difference that can be derived from FOCs. Note that we have

$$\left.\frac{d\hat{r}^R(x)}{dx}\right|_{x=1}=-3.$$

Furthermore, the derivatives of Δ^R are given by

$$\frac{\partial\Delta^R}{\partial r}=-\frac{5+x}{(5-2r+x)^2}<0;\quad \frac{\partial\Delta^R}{\partial x}=\frac{r}{(5-2r+x)^2}>0. \tag{A.6}$$

Then, a direct calculation shows that

$$\frac{\partial p_N^R}{\partial r} > 0,$$
$$\frac{\partial p_M^R}{\partial r} > 0,$$
$$\frac{\partial \Delta_M^R}{\partial r} < 0.$$

□

A.4. Proof of Proposition 2

We show that

$$p_M^P(x,0) - p_M^R(x,0) = \frac{2 - x - 3\Delta^P(x,0)}{2} - \frac{1}{1+x}$$

is positive. This expression can be written as

$$\frac{x(1-x) - 3(1+x)\Delta^P(x,0)}{2(1+x)}.$$

Noting that

$$\Delta^P(x,0) = \frac{7 - x - \sqrt{(7-x)^2 - 16x(1-x)}}{8},$$

and rearranging, we obtain

$$p_M^P(x,0) - p_M^R(x,0) = \frac{1}{16(1+x)}\left[3(1+x)\sqrt{(7-x)^2 - 16x(1-x)} - (5x^2 + 10x + 21)\right],$$

which is positive because

$$3(1+x)\sqrt{(7-x)^2 - 16x(1-x)} - (5x^2 + 10x + 21) \geq 0,$$

which is shown by the following inequality:

$$\begin{aligned}&9(1+x)^2[(7-x)^2 - 16x(1-x)] - (5x^2 + 10x + 21)^2\\ =&64x(1-x)(x+2)(3-2x) \geq 0.\end{aligned}$$

Next, we show that when x is large and $r = \bar{r} = x(1-x)/(2-x)$, $p_M^P < p_M^R$ holds. Because $\Delta^P(x, \bar{r}) = 0$, we have

$$p_M^P = p_N^P = \frac{2-x}{2}.$$

At $\bar{r}$, p_M^R is given by

$$p_M^R = \frac{1}{1+x} \frac{(2-x)(5+x) + (1-x^2)x}{(2-x)(5+x) - 2x(1-x)}.$$

Then, a calculation shows that

$$p_M^R - \frac{2-x}{2} = \frac{7x - 4 - x^2}{2(1+x)[(2-x)(5+x) + (1-x^2)x}(2-x)(5+x) - 2x(1-x)],$$

which is positive if and only if

$$x \geq \frac{7 - \sqrt{33}}{2}.$$

□

A.5. Proof of Proposition 3

We first show that at $r = 0$, $TS^P(\Delta^P, x) < TS^R(\Delta^R, x)$. Because $\Delta^P \in (0, x)$ when $r < \bar{r}$ and $TS^P(\Delta, x)$ in decreasing in Δ for $\Delta \geq 0$, we have $TS^P(0, x) > TS^P(\Delta^P, x)$. Furthermore, at $r = 0$, Δ^R holds, implying that

$$TS^R(\Delta^R, x) = TS^R(0, x) = TS^P(0, x) > TS^P(\Delta^P, x).$$

Next, we show that at $r = \bar{r}$, $TS^P(\Delta^P, x) > TS^R(\Delta^R, x)$. To see this, note that $\Delta^P = 0$ at $r = \bar{r}$, that $\Delta^R < 0$, and that $TS^R(\Delta, x)$ is increasing in Δ for all $\Delta \leq 0$. These imply that

$$TS^R(\Delta^R, x) < TS^R(0, x) = TS^P(0, x) = TS^P(\Delta^P, x).$$

Finally, we show that $TS^P(\Delta^P, x) - TS^R(\Delta^R, x)$ is increasing in r for $r \in [0, \bar{r}]$. This is shown by noting that

$$\begin{aligned}
\frac{\partial \Delta^R}{\partial r} &< 0, \\
\frac{\partial \Delta^P}{\partial r} &< 0, \\
\frac{\partial TS^P}{\partial \Delta} &< 0 \quad \text{for } \Delta \geq 0, \\
\frac{\partial TS^R}{\partial \Delta} &> 0 \quad \text{for } \Delta < 0,
\end{aligned}$$

which implies

$$\frac{\partial \left(TS^P(\Delta^P, x) - TS^R(\Delta^R, x)\right)}{\partial r} = \frac{\partial \Delta^P}{\partial r}\frac{\partial TS^P}{\partial \Delta} - \frac{\partial \Delta^R}{\partial r}\frac{\partial TS^R}{\partial \Delta} > 0.$$

Thus, there exists r^{TS} such that for all $r \in [0, r^{TS})$, $TS^P(\Delta^P, x) < TS^R(\Delta^R, x)$, and for all $r \in (r^{TS}, \bar{r}]$, $TS^P(\Delta^P, x) > TS^R(\Delta^R, x)$.

□

A.6. Proof of Proposition 4

First, we have

$$\begin{aligned}
\Delta CS =& CS^P(\Delta^P, p_M^P, x) - CS^R(\Delta^R, p_M^R, x) \\
=& \frac{2(1-\Delta^P)^3 - (1-\Delta^R)^3 - (1+\Delta^R)^3}{12} + \frac{(1-x)^2\Delta^P}{2} + \frac{\Delta^R}{2} - p_M^P + p_M^R \\
=& \frac{-3\Delta^P + 3(\Delta^P)^2 - (\Delta^P)^3 - 3(\Delta^R)^2}{6} + \frac{(1-x)^2\Delta^P}{2} + \frac{\Delta^R}{2} - p_M^P + p_M^R.
\end{aligned}$$

We can verify that when $\Delta^P = \Delta^R = \Delta \in (0, 1-x)$, and $p_M^P = p_M^R$, then $\Delta CS > 0$. By contrast, when $\Delta^P = \Delta^R = \Delta \in (x-1, 0)$, and $p_M^P = p_M^R$, then $\Delta CS < 0$.

At $r = 0$, $p_M^P \geq p_0 = p_M^R$, $\Delta^P > 0$, and $\Delta^R = 0$. Then,

$$\Delta CS \leq \frac{-\Delta^P(3(2-x)x - 3\Delta^P + (\Delta^P)^2)}{6} < 0,$$

where the last inequality follows from $\Delta^P \in (0, x)$. At $r = \bar{r}$, $\Delta^P = 0$ and $\Delta^R < 0$, yielding

$$\Delta CS = -\frac{(\Delta^R)^2}{2} + \frac{\Delta^R}{2} - p_M^P + p_M^R.$$

Furthermore, we have $p_M^P = 1 - x/2$ at $r = \bar{r}$, and

$$p_M^R = \frac{1}{1+x}\frac{5+x}{5+x-2r} - \Delta^R.$$

This further yields

$$\begin{aligned}
\Delta CS &= -\frac{\Delta^R}{2}(1+\Delta^R) - 1 + \frac{x}{2} + \frac{1}{1+x}\frac{5+x}{5+x-2\bar{r}} \\
&> -1 + \frac{x}{2} + \frac{1}{1+x}\frac{5+x}{5+x-2\bar{r}} \\
&= \frac{(1-x)}{2(1+x)(5+x-2\bar{r})(3-2x)}(22 - x + 5x^2 - 2x^3) > 0.
\end{aligned}$$

□

A.7. Proof of Proposition 5

Let $CS^P(\epsilon)$ and $CS^R(\epsilon)$ be the equilibrium consumer surplus under each search order. First, we have $CS^P(0) = CS^R(0)$ because search order does not matter when $x = 1$. Next, by Roy's identity and the fact that both sellers has the output $\frac{1}{2}$ when $x = 1$ and $r = 0$, we have

$$\begin{aligned}
CS^P(\epsilon) \simeq& CS^P(0) + \frac{\epsilon}{2}\left(\left.\frac{dp_N^P}{dx}\right|_{x=1,r=0} - \theta\left.\frac{dp_N^P}{dr}\right|_{x=1,r=0} + \left.\frac{dp_M^P}{dx}\right|_{x=1,r=0} - \theta\left.\frac{dp_M^P}{dr}\right|_{x=1,r=0}\right) \\
=& CS^P(0) - \frac{\epsilon}{2}\left(\frac{5+\theta}{12} + \frac{3+3\theta}{12}\right), \\
CS^R(\epsilon) \simeq& CS^R(0) + \frac{\epsilon}{2}\left(\left.\frac{dp_N^R}{dx}\right|_{x=1,r=0} - \theta\left.\frac{dp_N^R}{dr}\right|_{x=1,r=0} + \left.\frac{dp_M^R}{dx}\right|_{x=1,r=0} - \theta\left.\frac{dp_M^R}{dr}\right|_{x=1,r=0}\right) \\
=& CS^R(0) - \frac{\epsilon}{2}\left(\frac{3+2\theta}{12} + \frac{3+4\theta}{12}\right).
\end{aligned}$$

Thus, we have

$$\begin{aligned}
CS^P(\epsilon) - CS^R(\epsilon) &\simeq \frac{\epsilon}{2}\cdot\frac{1}{12}\left(3 + 2\theta + 3 + 4\theta - 5 - \theta - 3 - 3\theta\right) \\
&= \frac{\epsilon}{12}\left(-2 + 2\theta\right).
\end{aligned}$$

Therefore, $CS^P(\epsilon) > CS^R(\epsilon)$ if and only if $\theta > 1$ when $\epsilon \simeq 0$.

□

A.8. Proof of Proposition 6

Let

$$\Pi_M^k = D_M^k p_M^k + r D_N^k p_N^k$$

be the equilibrium profit of seller M for $k = P, R$. Noting that $D_M^k = 1/2$ and $p_M^k = 1/2$ at $\epsilon = 0$, we have

$$\Pi_M^k = \frac{1}{4} + \epsilon \frac{\partial \Pi^k}{\partial \epsilon}\bigg|_{\epsilon=0} + o(\epsilon),$$

where

$$\frac{\partial \Pi^k}{\partial \epsilon}\bigg|_{\epsilon=0} = \theta \frac{1}{4} + \frac{1}{2}\frac{\partial p_M^k}{\partial \epsilon} + \frac{\partial \Delta^k}{\partial \epsilon}\frac{\partial D_M^k}{\partial \Delta}.$$

A calculation shows that

$$\frac{\partial \Pi^P}{\partial \epsilon}\bigg|_{\epsilon=0} = \frac{\theta}{4} + \frac{3(1+\theta)}{24} + \frac{1-\theta}{6},$$
$$\frac{\partial \Pi^R}{\partial \epsilon}\bigg|_{\epsilon=0} = \frac{\theta}{4} + \frac{3+4\theta}{24} - \frac{\theta}{6},$$

implying that

$$\frac{\partial \Pi^P}{\partial \epsilon}\bigg|_{\epsilon=0} - \frac{\partial \Pi^R}{\partial \epsilon}\bigg|_{\epsilon=0} = \frac{4-\theta}{24} > 0$$

for all $\theta < \theta$. Thus, the platform always has an incentive to introduce search order distortion when search cost is small.

□

B. Omitted derivations

B.1. Derivation of total surplus functions

Consider a consumer who first visit seller 1 and then choose whether to search for seller 2. The surplus made by the transaction involving such a consumer is given by

$$\begin{aligned}
&\int_{x-\Delta_{21}}^{1} u du + \int_{0}^{x-\Delta_{21}} \left((u+\Delta_{21})u + \int_{u+\Delta_{21}}^{1} u_2 du_2 \right) du \\
=&\frac{1}{2} + \frac{(x-\Delta_{21})\left[x(3-2x)+\Delta_{21}(1-2\Delta_{21})\right]}{6} \\
=&\frac{1}{2} + \frac{x^2(3-2x)}{6} - \frac{\Delta_{21}^2(1+2x)}{6} + \frac{\Delta_{21}(\Delta_{21}^2 - x + x^2)}{3} \\
\equiv& ts(\Delta_{21}, x).
\end{aligned}$$

Then, the total surplus under search order distortion is given by

$$TS^P(\Delta, x) = ts(\Delta, x) = \frac{1}{2} + \frac{x^2(3-2x)}{6} - \frac{\Delta^2(1+2x)}{6} + \frac{\Delta(\Delta^2 - x + x^2)}{3},$$

and

$$TS^R(\Delta, x) = \frac{ts(\Delta, x) + ts(-\Delta, x)}{2} = \frac{1}{2} + \frac{x^2(3-2x)}{6} - \frac{\Delta^2(1+2x)}{6}.$$

We have

$$\frac{\partial TS^P(\Delta, x)}{\partial \Delta} = -\frac{(3\Delta + 1 - x)(\Delta + x)}{3},$$

which is negative as long as $\Delta \geq \max\{-x, -(1-x)/3\}$. We also have

$$\frac{\partial TS^R(\Delta, x)}{\partial \Delta} = -\frac{\Delta(1+2x)}{3},$$

which is positive as long as $\Delta < 0$.

B.2. Derivation of consumer surplus function

$$
\begin{aligned}
&\int_{x-\Delta_{21}}^{1}(u-p_1)du+\int_{0}^{x-\Delta_{21}}\left((u+\Delta_{21})(u-p_1)+\int_{u+\Delta_{21}}^{1}(u_2-p_2)du_2-s\right)du\\
=&\frac{1-(x-\Delta_{21})^2}{2}-(x-\Delta_{21})s-p_1\\
&+\frac{1}{2}\left((x-\Delta_{21})(1-\Delta_{21})^2+\Delta_{21}(x-\Delta_{21})^2+\frac{(x-\Delta_{21})^3}{3}\right)\\
=&\frac{1}{2}+\frac{(1-x)(x-\Delta_{21})^2}{2}+\frac{(x-\Delta_{21})^3}{6}-p_1\\
=&\frac{1}{2}-\frac{(1-x)^3}{6}-\frac{(1-x)^2(x-\Delta_{21})}{2}+\frac{(1-\Delta_{21})^3}{6}-p_1\\
\equiv&cs(\Delta_{21},p_1,x).
\end{aligned}
$$

We have

$$
\begin{aligned}
CS^P(\Delta,p_M,x)&=cs(\Delta,p_M,x)\\
&=\frac{1}{2}-\frac{(1-x)^3}{6}-\frac{(1-x)^2x}{2}+\frac{(1-x)^2\Delta}{2}+\frac{(1-\Delta)^3}{6}-p_M,
\end{aligned}
$$

and

$$
\begin{aligned}
CS^R(\Delta,p_M,x)&=\frac{cs(\Delta,p_M,x)+cs(-\Delta,p_M+\Delta,x)}{2}\\
&=\frac{1}{2}-\frac{(1-x)^3}{6}-\frac{(1-x)^2x}{2}+\frac{(1-\Delta)^3}{12}+\frac{(1+\Delta)^3}{12}-p_M-\frac{\Delta}{2}.
\end{aligned}
$$

B.3. Comparison of and reaction functions of seller M

By arranging the first-order condition of the platform under random search, (A.3), its reaction function can be written as

$$br^R(p_N,x,r)=\frac{1+[2+(1+x)r]p_N}{3+x}.$$

By arranging the first-order condition for the platform under search order distortion, (4), its reaction function can be written as

$$br^P(p_N,x,r)=\frac{-[2-(2+r)p_N]+\sqrt{X(p_N,x,r)}}{3},$$

where

$$X(p_N, x, r) = [2-(2+r)p_N]^2 + 12[2-2x+x^2+p_N[2+2r-(1+2r)p_N].$$

Then, we have

$$\frac{\partial br^R}{\partial r} = \frac{1+x}{3+x}p_N,$$

and

$$\frac{\partial br^P}{\partial r} = \frac{p_N}{3}\left(1-\frac{2-(2-r)p_N+12p_N^2}{\sqrt{X(p_N,x,r)}}\right).$$

Because we have

$$\frac{1+x}{3+x}p_N - \frac{p_N}{3}\left(1-\frac{2-(2-r)p_N+12p_N^2}{\sqrt{X(p_N,x,r)}}\right) = \left(\frac{2x}{3(3+x)}+\frac{2-(2-r)p_N+12p_N^2}{3\sqrt{X(p_N,x,r)}}\right)p_N > 0,$$

we have $\partial br^R/\partial r > \partial br^P/\partial r$, which implies that the platform responds to the increase in commission rate under random search more than it does under search order distortion.

B.4. Approximate values of equilibrium variables with small search costs

First, we compute the approximated values of equilibrium price differences. Because we have

$$\Delta^P(1,r) = \frac{3-r-\sqrt{(3-r)^2+(4-r)r}}{4-r}, \text{ and } \Delta^P(x,0) = \frac{7-x-\sqrt{(7-x)^2-16x(1-x)}}{8},$$

we have

$$\frac{\partial \Delta^P(1,0)}{\partial r} = -\frac{1}{6}, \text{ and } \frac{\partial \Delta^P(1,0)}{\partial x} = -\frac{1}{6}.$$

We also have

$$\Delta^R(1,r) = -\frac{1}{2}\frac{r}{3-r}, \text{ and } \Delta^R(x,0) = 0.$$

Thus, we have

$$\frac{\partial \Delta^R(1,0)}{\partial r} = -\frac{1}{6},$$

and

$$\frac{\partial \Delta^R(1,0)}{\partial x} = 0.$$

Suppose that $x = 1 - \epsilon$ and $r = \theta\epsilon$ for sufficiently small ϵ and $\theta \leq 2$. Then, we have

$$\Delta^P \simeq \epsilon \frac{1-\theta}{6}, \text{ and } \Delta^R \simeq -\epsilon \frac{\theta}{6}.$$

Using the formula for $p_N^R, p_M^R, p_N^P, p_M^P$, we have

$$\left.\frac{dp_N^P}{dx}\right|_{x=1,r=0} = -\frac{5}{12}, \quad \left.\frac{dp_M^P}{dx}\right|_{x=1,r=0} = -\frac{1}{4},$$
$$\left.\frac{dp_N^R}{dx}\right|_{x=1,r=0} = \left.\frac{dp_M^R}{dx}\right|_{x=1,r=0} = -\frac{1}{4}.$$

We also have

$$\left.\frac{dp_N^P}{dr}\right|_{x=1,r=0} = \frac{1}{12}, \quad \left.\frac{dp_M^P}{dr}\right|_{x=1,r=0} = \frac{1}{4},$$
$$\left.\frac{dp_N^R}{dr}\right|_{x=1,r=0} = \frac{1}{6}, \quad \left.\frac{dp_M^R}{dr}\right|_{x=1,r=0} = \frac{1}{3}.$$

Thus, if $x = 1 - \epsilon$ and $r = \theta\epsilon$ for sufficiently small ϵ, we have

$$p_N^P = \frac{1}{2} + \epsilon\left(\frac{5+\theta}{12}\right) + o(\epsilon), \quad p_M^P = \frac{1}{2} + \epsilon\left(\frac{3+3\theta}{12}\right) + o(\epsilon),$$
$$p_N^R = \frac{1}{2} + \epsilon\left(\frac{3+2\theta}{12}\right) + o(\epsilon), \quad p_M^R = \frac{1}{2} + \epsilon\left(\frac{3+4\theta}{12}\right) + o(\epsilon).$$

Finally, note that we have

$$\left.\frac{d\hat{r}^P(x)}{dx}\right|_{x=1} = -7, \text{ and } \left.\frac{d\hat{r}^R(x)}{dx}\right|_{x=1} = -3.$$

Hence, if $x = 1 - \epsilon$ for sufficiently small $\epsilon > 0$, $\min\{r^P(x), r^R(x)\} = 3\epsilon + o(\epsilon)$. Therefore, our analysis around $\epsilon = 0$ is valid as long as $r = \theta\epsilon$ for $\theta \in [0, 3)$.